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DYNAMICS OF LIQUID-FILLED SHELL: RESONANCE IN MODIFIED CYLINDRICAL CAVITIES

by

B. G. Karpov

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ABSTRACT

A search was made, using a liquid-filled gyroscope, for resonance frequencies in modified cylindrical cavities. The fill-ratio at which the resonant frequency occurs shows considerable sensitivity to cylindrical modifications. Simple rules are given for predicting, approximately, the change in this fill-ratio as a function of the modifications. In the Appendix, Wedemeyer suggests a better way of prediction.

Tests of a cylindrical cavity with a central rod show that the air gap between the column and the free surface of the disturbed fluid should not, at small yaws, be less than 1/16 inch for non-interference with resonant conditions.

TABLE OF CONTENTS

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1. INTRODUCTION

For a liquid-filled shell having a cylindrical cavity, Stewartson's theory[1*] with Wedemeyer's viscous correction[2] gives excellent prediction, over a wide range of Reynolds numbers, of the fill-ratios at which resonance between fluid frequencies and the shell nutational frequency occurs[3,4]. As is well known, such coincidence of frequencies may lead to dynamic instability of the liquid-filled shell.

Unfortunately, however, most shell cavities are not cylindrical. The shell are usually streamlined and the shape of the cavity, in general, follows the outer form. It is not possible to predict the dynamic behavior of liquid-filled shell with noncylindrical cavities. The difficulty is fundamental. The theoretical formulation of the hydrodynamical problem is the same as for the cylindrical cavity, but because of the nature of the boundary conditions no full analytical solution, as available for the cylindrical cavity, seems accessible. The problem is very difficult even for high speed computers.

To shed some light on the behavior of the eigen-frequencies for modified cylindrical cavities, the experiments to be reported here were undertaken. Because of the great many ways in which a cylindrical cavity may be deformed, these limited experiments are only exploratory. Therefore, in the absence of theoretical guide lines, the results may be of limited usefulness and could not be broadly generalized. Nevertheless, in order to be of some use to the designer, an attempt was made to study only those changes in the geometry of the cylindrical cavity that approximated the shapes of the cavities of the actual shell. It

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**Superscript numbers denote references which may be found on page* 39.

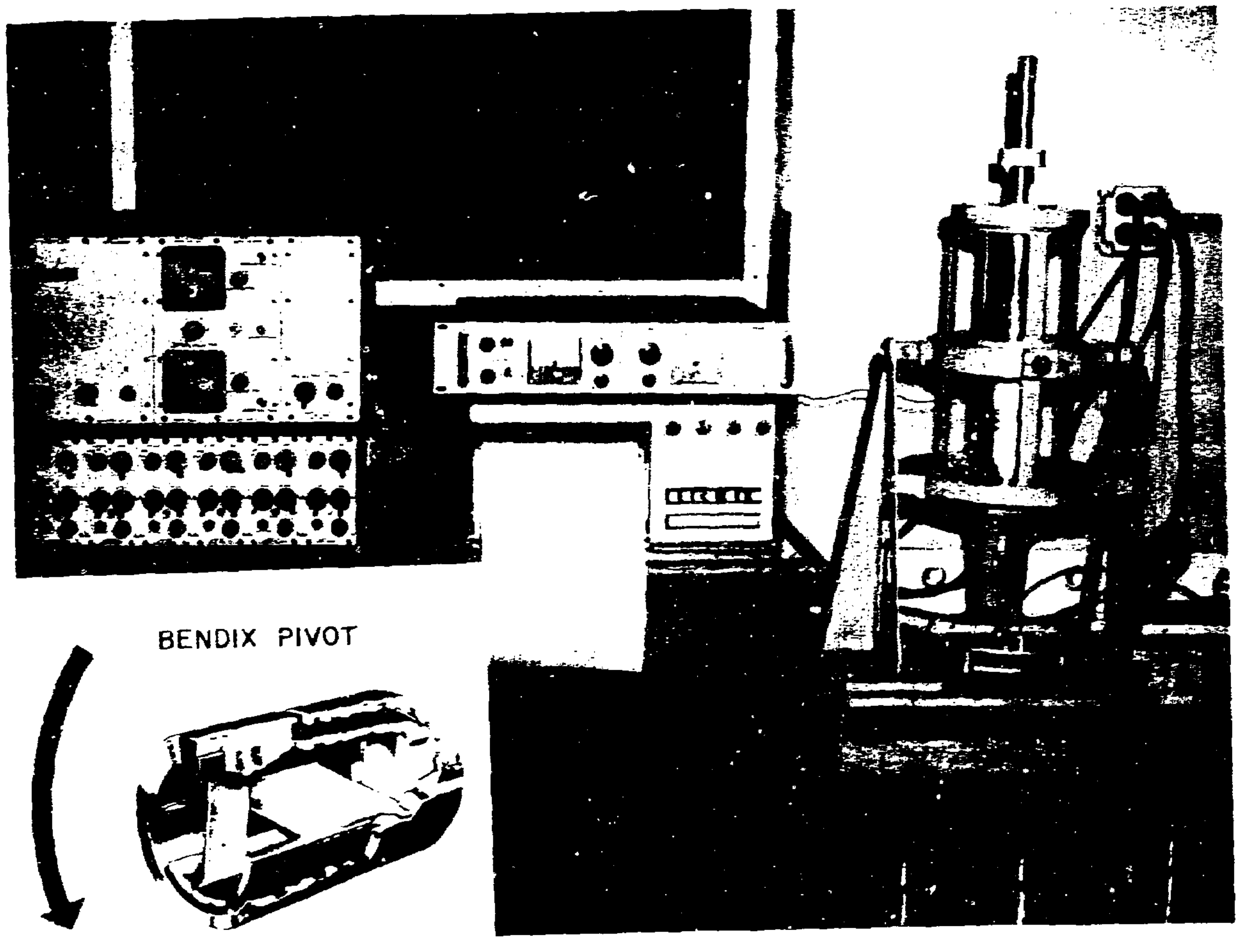

FIGURE 1. GYROSCOPE

was hoped that the behavior of fluid frequencies in modified cavities could be related to their behavior in an easily established "equivalent cylinder" determined, perhaps, from the volume of a modified cavity. If this could be accomplished, then Stewartson's tables could be used for practical prediction of resonance in non-cylindrical cavities.

Subsequent to completion of the present experiments, Wedemeyer developed an analysis for predicting eigen-frequencies in non-cylindrical cavities through a perturbation of the cylindrical cavity solution. The restriction of this analysis is that the cavity geometry deviate only mildly (e.g., cavity radius changes slowly as a function of cavity length) from that of a pure cylinder.* Data presented in the present report that satisfy this restriction are in good agreement with the results of this analysis.

2. THE GYROSCOPE

The experimental tool was the gyroscope. It is shown in Figure 1 and was described in some detail in Reference 4. Its principal characteristic is that its inner and outer gimbals are supported by frictionless bearings. Its sensitivity is high and it gives reproducible outputs at amplitudes of less than 1°. The gyroscope's oscillations are recorded through strain gages mounted on the pivots and are monitored on a photographic recorder. It is balanced about the pivot point; hence, its motion is essentially pure nutation. The rotor contains a lucite cylinder whose ends or "cups" can be readily modified to various desired geometries.

*A summary of this development appears as Appendix A of this report. The full analysis appears in "Dynamics of Liquid-Filled Shell: Non-Cylindrical Cavity", by E. H. Wedemeyer, BRL R 1326 August 1966.

The non-dimensional nutational frequency of this liquid-filled system, τ_n, is .055. Various cavity geometries produced changes in this frequency only in the fourth decimal and, therefore, they were neglected.

The fluid was a silicon oil of viscosity one centistoke and of specific gravity 0.818 at 77°F. The gyroscope was spun at 5000 rpm leading to a Reynolds number for the experiments of 5.19×10^5 = $Re = \frac{\omega a^2}{\nu}$ where ω = spin, a = radius of the cavity, and ν = viscosity of the fluid.

As a side issue, an investigation was made of the interference between a central column and the free surface of the disturbed fluid in a resonating cylindrical cavity. The central column simulated a "burster", the explosive device frequently used in liquid-filled shell. The search for resonance in such a configuration by use of Stewartson's tables is valid only if the central column does not interfere with the oscillations of the free surface of the fluid. Therefore, it is necessary to know the minimum air gap between the column and the fluid so that this condition is satisfied.

The results of these various experiments are the subject of the present report.

3. THE EXPERIMENTS

3.1 The Effect of Rounded Corners

The original cylinder, used in Reference 4, was 2c = 7.630" in height and 2a = 2.48" in diameter or of fineness ratio c/a = 3.077. It was designed to resonate with the principal fluid frequency, j= 1, at 85 percent fill-ratio. This fineness ratio was kept constant while the ends of the cylinder were modified by rounding the corners to various radii as is shown in Figure 2. The results of the tests are summarized in Table I and shown graphically in Figure 3.

Relative to the original cylindrical volume, the fill-ratio at which resonance occurs, $\tau_o = \tau_n$, changes relatively little up to R/a = 0.60. For larger radii the resonance occurs at progressively smaller fill-ratios, and for hemispherical ends, R/a = 1, the fill-ratio at which resonance occurs is 13 percent lower. For such a cavity, an "equivalent cylinder" is of smaller fineness ratio.

The instability of such configurations, as measured by the rate of divergence of the nutational amplitude, α_1, decreases progressively as R/a increases, and for R/a = 1 at both ends, the instability is only 55 percent of its value for the unmodified cylinder.

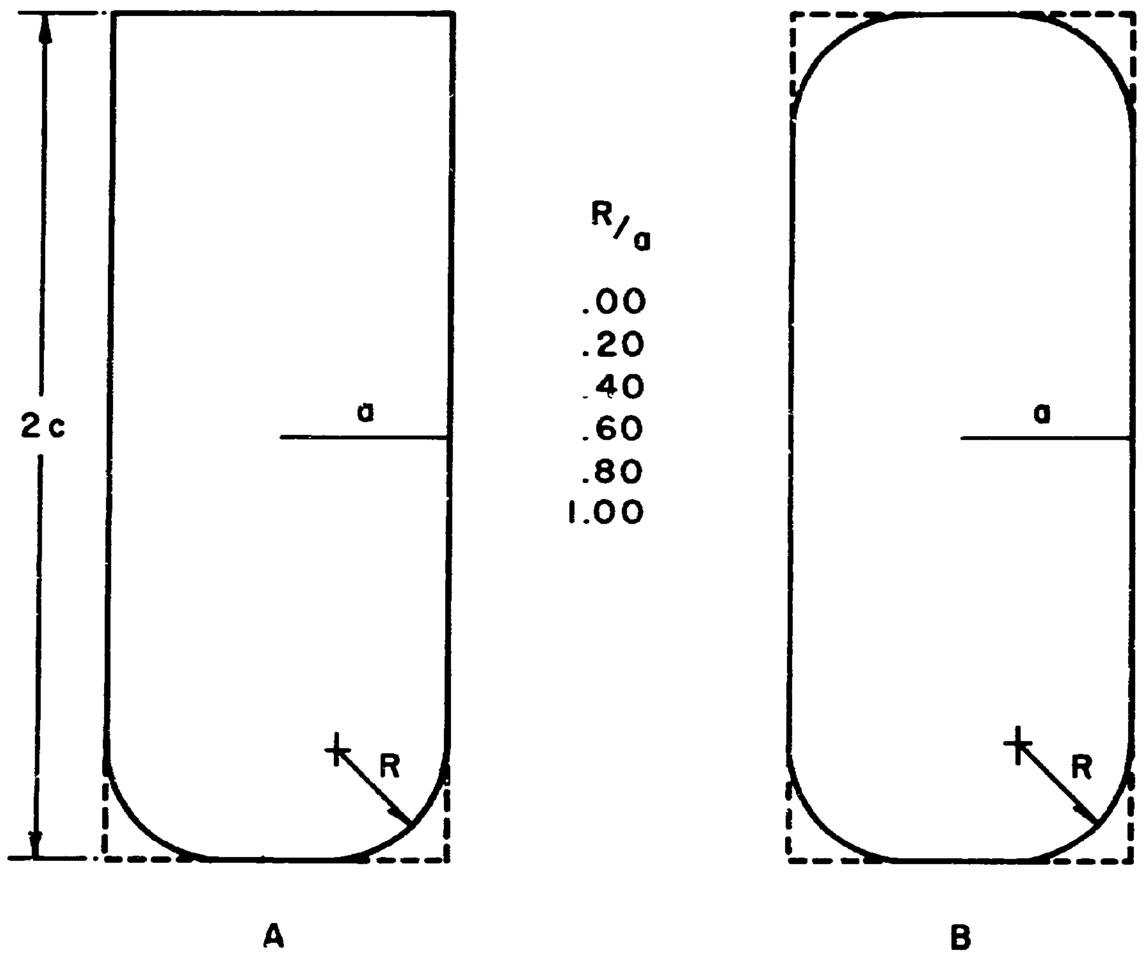

FIGURE 2. MODIFIED CYLINDRICAL CAVITIES

TABLE I

Effect of Rounded Corners On Resonance

Fill-Ratio for Resonance $\tau_o = \tau_n = .055$

$c/a = 3.077$ $(j = 1)$ $\nu = 1$ c.s. 5000 rpm.

Volume of right cylinder $V_c = 605$ cc.

Configuration A

R/a	V_o 100% cc	V Resonance cc	100 V/V_c	α_1 per sec.
.00	605	511	84.5	.522
.20	-	-	-	-
.40	598	513	84.8	.500
.60	592	513	84.8	.460
.80	584	497	82.1	.440
1.00	574	476	78.7	.399

Configuration B

R/a	V_o 100% cc	V Resonance cc	100 V/V_c	α_1 per sec.
.00	605	511	84.5	.522
.20	600	510	84.4	.535
.40	590	512	84.6	.495
.60	575	506	83.6	.470
.80	550	474	78.3	.350
1.00	532	434	71.7	.288

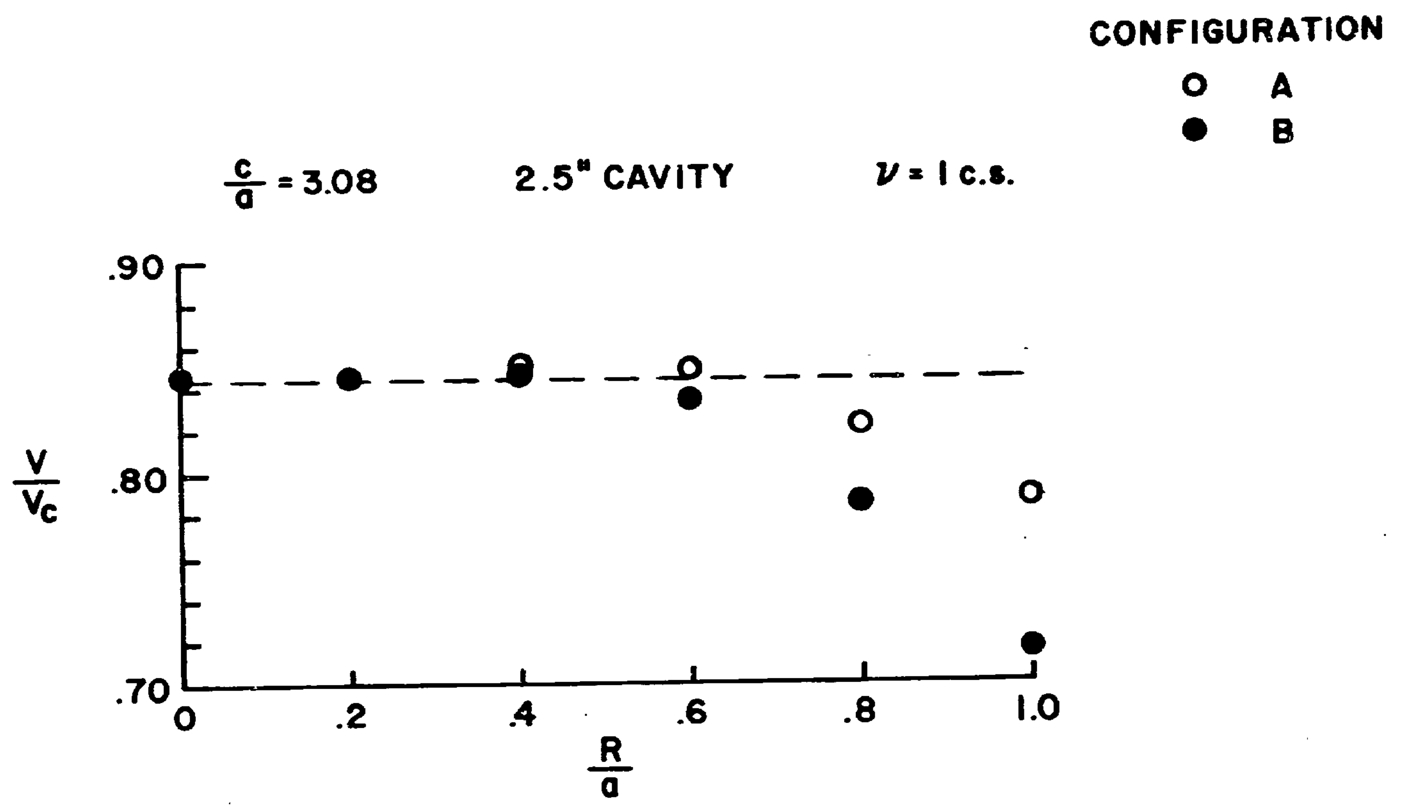

FIGURE 3. EFFECT OF ROUNDING CORNERS, RESONANCE AT $\frac{V}{V_c}$

3.2 The Effect of Conical Cups

The geometrics tested are shown in Figure 4. Various cups are defined by two parameters: the angle θ, and the height h or by h/2c where 2c is the height of the cylinder. Initially, the tests were run in modified cylinders of fineness ratio $c/a = 3.077$. In an unmodified cylinder of this fineness ratio, resonance occurs at 85 percent fill-ratio. When it was discovered that the change in the fill-ratio, to obtain resonance in modified cylinders, was larger than the available 15 percent, from 85 to 100 percent full, a new cylindrical cavity was designed to resonate at 60 percent. The fineness ratio of this cavity was $c/a = 2.687$.

The results are shown in Figure 5 where the reference volume is the actual 100 percent full volume of the modified cavity, V_{oi}. The resonant frequency, $\tau_o = \tau_n$, occurs at markedly different fill-ratios; the longer the conical cup, or the greater the angle, the greater is the change in the fill-ratio. Figure 6 shows the same data with reference to the volume of the unmodified cylinder. As was the case with rounded corners, the greater the departure from the cylindrical cavity, the smaller the instability. The actual data are presented in Tables II and III.

Table IV summarizes the data from other tests, such as: combinations of a conical cup with a rounded base, (configurations F_1 and F_2, see Figure 4); a cup with a radius of curvature, E_2; and a combination of two conical cups, C and E_1. The discussion of these results will be deferred to a later section. It suffices here to say that the trend of the variations in the fill-ratios for various combinations is consistent

with the results shown in Tables I, II, and III, i.e., the effects of combinations of modifications are, approximately, additive.

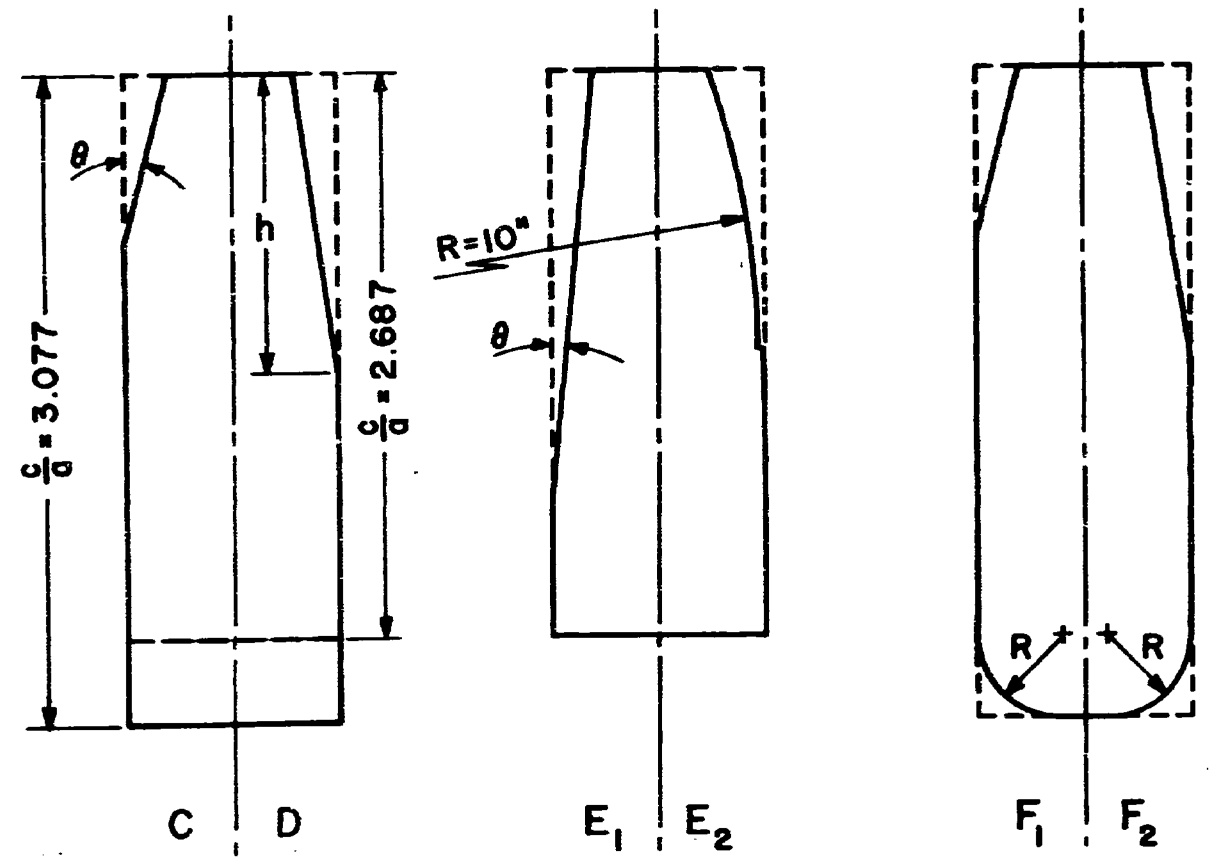

FIGURE 4. CONFIGURATIONS TESTED

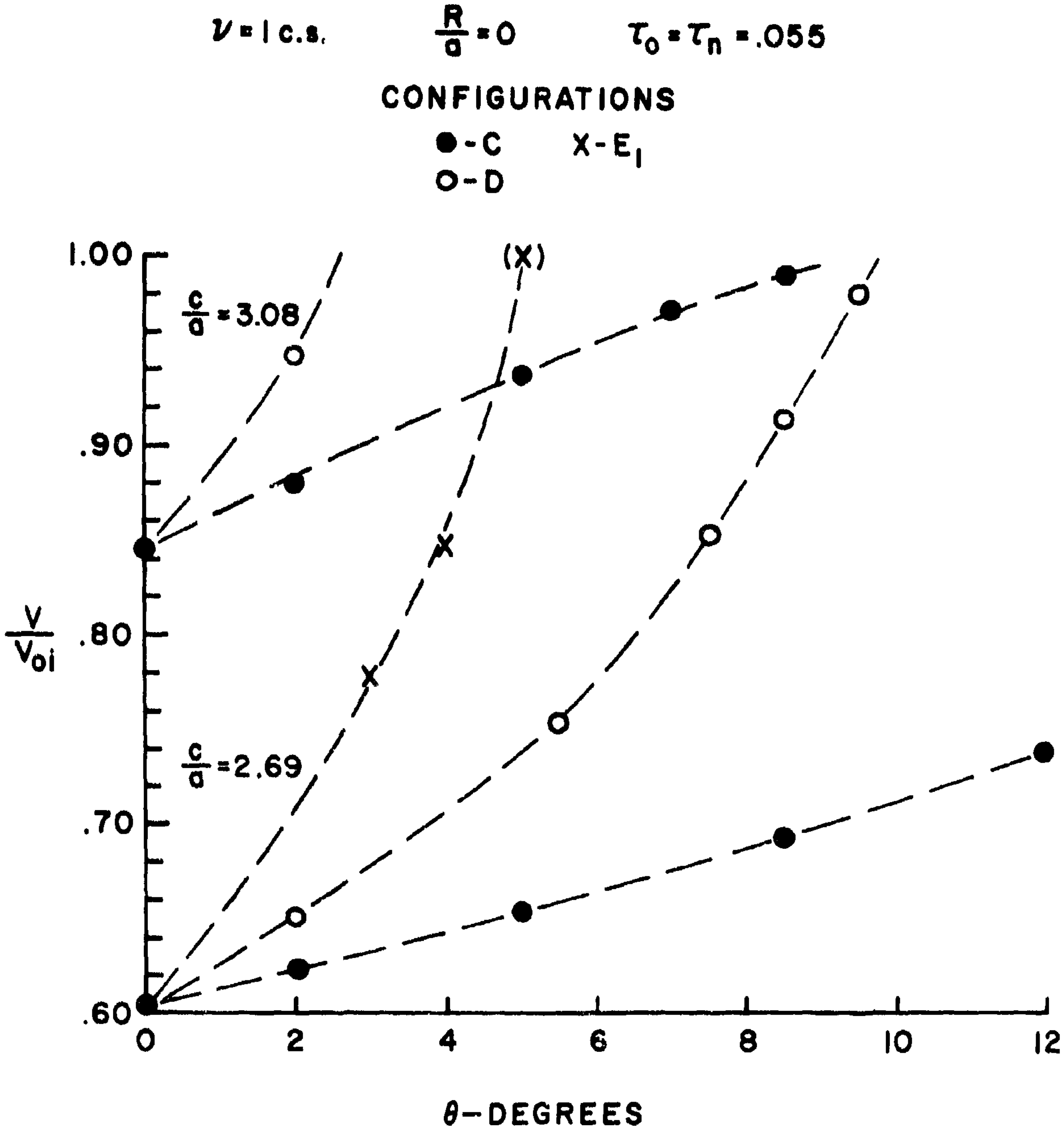

FIGURE 5. EFFECT OF CONICAL CUPS, RESONANCE AT $\frac{V}{V_{oi}}$

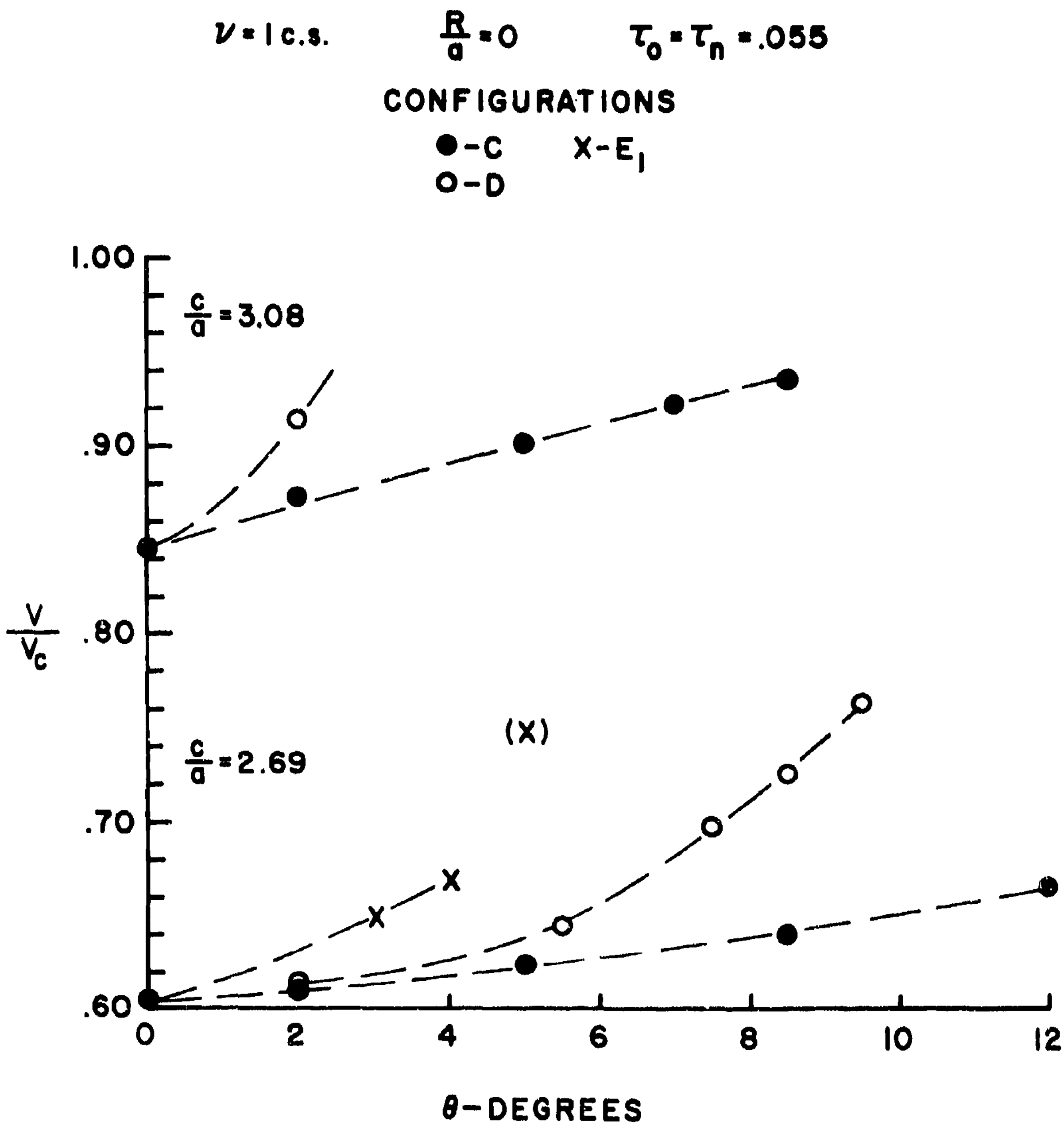

FIGURE 6. EFFECT OF CONICAL CUPS, RESONANCE AT $\frac{V}{V_c}$

TABLE II

Effect of Conical Cups on Resonance

$c/a = 3.077$ $\qquad$ $R/a = 0$ $\qquad$ $\tau_o = \tau_n = .055$

Configuration C

$\frac{h}{2c} = .26$

θ Degrees	V_{oi} 100% full cc	V Resonance cc	$100V/V_{oi}$	$100V/V_c$	α_1 per sec.
0	605	511	84.5	84.5	.522
2*	600	528	88.0	87.3	.484
5	582	546	93.8	90.2	.405
7	578	558	96.5	92.2	.333
8½	572	566	99.0	93.6	.188

Configuration D

$\frac{h}{2c} = .46$

θ Degrees	V_{oi} 100% full cc	V Resonance cc	$100V/V_{oi}$	$100V/V_c$	α_1 per sec.
0	605	511	84.5	84.5	.522
2	582	552	94.8	91.5	.415

*Apparent inconsistencies in volumes, V_{oi}, for 2^o cup, which may be detected in subsequent tables, arose from slight modification of this cup. The lip was made thinner thereby increasing the total volume by about 12 cc.

TABLE III

c/a = 2.687   R/a = 0

Configuration C

$\frac{h}{2c} = .30$

θ	V_{oi} cc	V cc	$100V/V_{oi}$	$100V/V_c$	α_1 per sec.
0	534	322	60.3	60.3	.364
2	522	326	62.5	61.0	.322
5	508	332	65.4	62.2	.282
$8\frac{1}{2}$	494	342	69.2	64.0	.195
12	482	356	73.9	66.7	.140

Configuration D

$\frac{h}{2c} = .53$

θ	V_{oi} cc	V cc	$100V/V_{oi}$	$100V/V_c$	α_1 per sec.
2	502	326	64.9	61.0	.311
$5\frac{1}{2}$	457	344	75.3	64.4	.220
$7\frac{1}{2}$	436	373	85.3	69.8	.130
$8\frac{1}{2}$	424	388	91.5	72.7	.135
$9\frac{1}{2}$	416	408	98.1	76.4	.137

Configuration E_1

$\frac{h}{2c} = .75$

θ	V_{oi} cc	V cc	$100V/V_{oi}$	$100V/V_c$	α_1 per sec.
3	445	347	77.9	65.0	.318
4	422	358	84.8	67.0	.242
5	398	398	100:	75:	.065:

TABLE IV

Effect of a Conical Cup and a Rounded Base

$c/a = 3.077$ $\tau_o = \tau_n = .055$ $V_c = 605$ cc

Configuration F_1

R/a	V_{oi} cc	V_1 cc Main Resonance ($\frac{h}{2c} = .26$)	V_2 cc	$100 \frac{V_1}{V_c}$ ($\theta = 2^o$)	$100 \frac{V_2}{V_c}$	α_1 per sec	α_2 per sec	Remarks
.00	588	528	-	87.3	-	.484	-	
.40	578	538	-	88.9	-	.390	-	
.60	570	534	542	88.2	89.6	.340	.245*	Double peak
.80	560	514	-	85.0	-	.280		

$c/a = 2.687$

Configuration F_2

$\frac{h}{2c} = .53$ $\theta = 2^o$ $V_c = 534$ cc

R/a	V_{oi} cc	V_1 cc	V_2 cc	$100 \frac{V_1}{V_c}$	$100 \frac{V_2}{V_c}$	α_1 per sec	α_2 per sec
.00	522	326	-	61.0	-	.322	
.40	482	324	382	60.7	62.2	.225	.100*
.60	470	308	318	57.7	59.6	.205	.115*
.80	460	294	304	55.0	56.9	.210	.070*

$\theta = 5\frac{1}{2}^o$

R/a	V_{oi} cc	V_1 cc	V_2 cc	$100 \frac{V_1}{V_c}$	$100 \frac{V_2}{V_c}$	α_1 per sec	α_2 per sec
.00	457	344	336	64.4	62.9	.220	.045*
.40	450	340	-	63.7	-	.240	
.60	442	334	-	62.5	-	.200	
.80	432	321	-	60.1	-	.160	

TABLE IV (continued)

Configuration E_2

Circular Arc of Radius 10"

$c/a = 2.551$ $V_c = 501$ cc

$\frac{h}{2c} = .48$

R/a	V_{oi} cc	V_1 cc	V_2 cc	$100\frac{V_1}{V_c}$	$100\frac{V_2}{V_c}$	α_1 per sec	α_2 per sec	Remarks
.00	390	346	354	69.1	70.7	.105	.050	
.40	384	338	344	67.5	68.7	.080	.075	
.60	374	331	-	66.1	-	.070	-	very broad
.80	364	317	-	63.3	-	.090	-	

Combination Cups

$c/a = 2.687$ $V_c = 534$ cc

Cup C together with Cup E_1

Test (a)

Cups	θ^o	V_{o_1} cc	V_1cc	V_2	$\frac{V_1}{V_c}$	$\frac{V_2}{V_c}$	α_1	α_2	Remarks
C	7	-	-	-	-	-	-	-	
E_1	3	415	342	358	64.0	67.0	.132	.144	Two Peaks
				Text (b)					
C	$8\frac{1}{2}$								
E_1	3	407	358	-	67.0	-	.109	-	

NOTE: Cup E_1, $\frac{h}{2c} = .75$, was cut to $\frac{h}{2c} = .70$ so as to add, with Cup C, $\frac{h}{2c} = .30$, to unity.

3.3 The Effect of a Central Rod on Resonance

In practice, not infrequently the liquid-filled shell cavity contains a central column or "burster" and the available volume is then filled to some fraction, say, β. Stewartson's tables can be used to search for resonance in such a cavity provided that the column does not interfere with the liquid. For a completely filled cavity of this nature, Stewartson's theory is not applicable because of the presence of a solid inner boundary not considered by Stewartson.

If, therefore, the available volume of the cavity with a burster is filled to β fraction, then the air fraction relative to the cavity without the burster is:

$$b^2/a^2 = 1 - \beta(1 - r^2/a^2)$$

where 2b is the diameter of the air column and 2r is the diameter of the burster. For non-interference, therefore, we are interested in finding a minimum air gap, b - r, which will satisfy this condition.

For this purpose the following experiments were performed. The cylindrical cavity, c/a = 3.077, was used. Its volume is 605 cc, and it resonated at 85 percent fill-ratio, or with 511 cc of liquid. The diameter of the air column at resonance was 0.98 inch. When a plastic central rod of this diameter was introduced into the cavity, thus filling the air space, the resonance was suppressed. Maintaining the same amount of liquid in the cavity, 511 cc, the diameter of the central rod was progressively reduced, in steps of 1/32 inch, until a fairly full recovery of the original resonance was achieved. The results of these experiments are shown in Figure 7 in the form of instability as measured by the rate

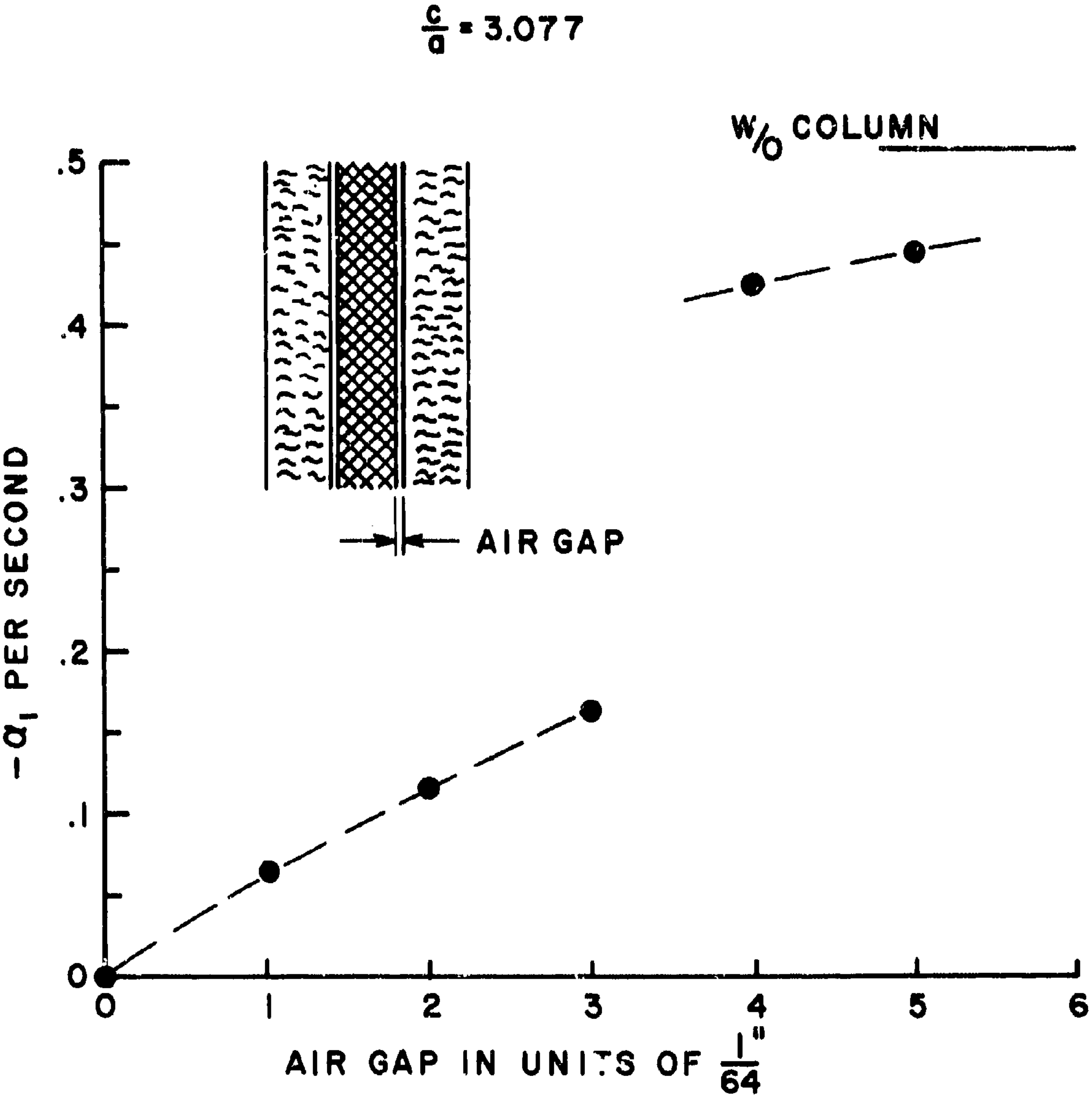

FIGURE 7. INTERFERENCE OF CENTRAL COLUMN WITH RESONANCE

of divergence of the nutation amplitude, $e^{\alpha_1 t}$, versus air gap.

The results show that in order to assure non-interference between the central column and the disturbed free surface of the liquid at resonance, the air gap (b - r) should not be less than 1/16 inch. Expressed in another way, the fill-ratio relative to the available volume should not be greater than 96 percent for valid use of Stewartson's tables for this type of cavity. This rule should apply at least at relatively small yaws. The amplitudes of oscillations of the free surface will grow with the larger yaws.

At gyroscopic spin of 5000 rpm, parabolic departure of the free surface from cylindrical is of the same order of magnitude as the air gap. It can be easily shown that the difference in radii of the air cavity at the top, r_2, and at the bottom, r_1 is approximately:

$$r_2 - r_1 \simeq \frac{2g}{\omega^2} \left(\frac{c}{a}\right) \frac{1}{\sqrt{\alpha}}$$

where α is the fraction of the total volume occupied by air. In our case, $\alpha = 0.15$ and $r_2 - r_1 = \frac{1}{64}$ inch, whereas the transition to "non-interference" occured at $\frac{4}{64}$ inch. Thus, the air gap at the bottom of the cavity is somewhat less than at the top. At higher shell spins, however, this difference becomes negligibly small or at lower spins increacingly significant. Thus, the above limitations on the air gap is fairly rough, and in practice, should be enlarged.

4. DISCUSSION

Present experiments show that modification of a cylindrical cavity causes a substantial shift in the fluid frequencies. The same fluid frequency, in a modified cavity, occurs either at lower or higher fill-ratio than in a corresponding cylindrical cavity of the same fineness ratio, c/a. Rounding the corners of a cylindrical cavity, for example, shifts the frequencies to a lower fill-ratio; introducing a conical constriction at the end of the cylinder shifts the frequencies to higher fill-ratios. This is illustrated, schematically, in Figure 8. A solid curve on the frequency-fill-ratio diagram is taken from Stewartson's tables for a cylindrical cavity of a specified fineness ratio. Dashed curves are the conjectured shifts of the frequencies in modified cavities. The amount and the nature of the shift is related to the kind and the degree of modification. The dashed curves are drawn for conical cups. It is seen that at the same fill-ratio the frequencies are higher, or the same frequency, such as the resonant frequency $\tau_o = \tau_n$, occurs at a higher fill-ratio. For rounded corners the effect seems to be in the opposite direction, the frequencies are lowered and the dashed curve should be drawn under the solid curve. Thus, the resonant frequency would occur at a lower fill-ratio.

From Stewartson's tables one can draw also the fineness-ratio versus the fill-ratio diagram for a particular frequency. Such a diagram would show that an increase in the fill-ratio corresponds to an effective increase in the fineness ratio and vice versa.

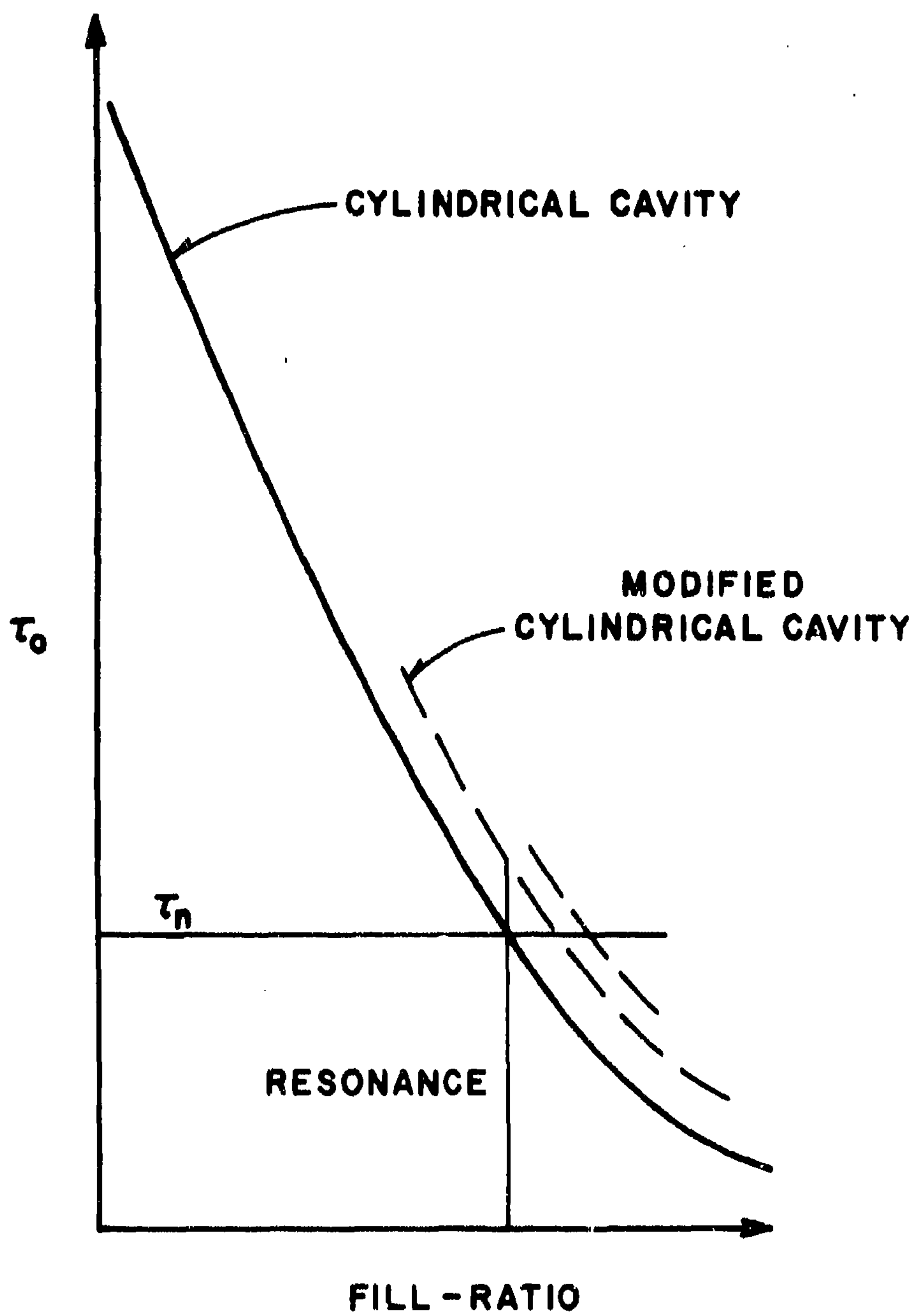

FIGURE 8. SCHEMATIC OF CHANGE OF τ_0

4.1 Rounding Corners

Table I shows that in a cylindrical cavity of $c/a = 3.08$, the effect of rounded corners begins to be significant for $R/a \geq 0.8$. At larger radii, the resonance occurs at a lower fill-ratio, corresponding to a smaller "effective" fineness ratio. One may define, therefore, an "equivalent cylindrical cavity" of diameter 2a but of reduced height in a ratio of modified to unmodified volumes. If the volume of the modified cavity is represented as:

$$V' = \pi a^2 \, 2c', \tag{1}$$

the fineness ratio of an "equivalent cylinder" will be

$$c'/a = c/a \left(\frac{V'}{V_c}\right) \tag{2}$$

where V_c is the volume of the unmodified cylinder. The corresponding fill-ratio for resonance with the new c'/a is easily found from Stewartson's tables. The results of such an exercise are shown in Table V.

TABLE V

Prediction of Effect Due to Rounded Corners by an "Equivalent Cylindrical Cavity", ECC

	Cylinder $c/a = 3.077$ Configuration A		$V_c = 605$ cc Configuration B	
R/a	0.8	1.0	0.8	1.0
100% vol., cc	584	574	550	532
Obs. resonance, % fill	82.1	78.7	78.3	71.7
Obs. $(c/a)_{eff}$	3.039	3.009	3.003	2.901
Computed c'/a	2.970	2.919	2.796	2.704
Predicted resonance, % fill	76	73	65	61
Diff., (O - C), % fill	6	6	13	11

Clearly, this simple device does not work very well. With large rounded corners the frequencies change as if an "equivalent cylinder" is both shorter and narrower so as to increase the c'/a, as computed above, to the observed $(c/a)_{eff}$. Of course, it is possible to devise an appropriate change in both the height, 2c, and the diameter, 2a, so as to satisfy the observations. But such arbitrary manipulation is not warranted. Suffice here to say that prediction of the effect of rounding corners for $R/a \geq 0.8$, by the method outlined above, overestimates the reduction in the fill-ratio by 6 percent for configuration A and by 12 percent for configuration B. The effects appear to be approximately additive. For practical purposes, therefore, prediction of resonance in a cylindrical cavity with rounded corners can be made by ignoring the effect if $R/a \leq .6$; for larger radii, one may estimate the effect either from Figure 2 or by an "equivalent cylinder" scheme outlined above, i.e., compute the fill-ratio from an effective fineness ratio $c'/a = c/a(V'/V_c)$ and correct the resulting fill-ratio by the above percentages.

4.2 Conical Cups

Experiments show that conical cups, as shown in Figure 4, can change markedly the fluid frequencies relative to those in an unmodified cylinder of the same height, 2c, and of the same diameter, 2a. In a modified cylinder the "effective" cavity appears to be of greater fineness ratio than the corresponding cylinder. Or, expressing it in another way, a given frequency, such as resonant frequency $\tau_o = \tau_n$, appears in a modified cylinder at a higher fill-ratio than in an unmodified cylinder.

One may try, therefore, to define an "equivalent cylinder" as having the volume of a modified cylinder, of the original height 2c, but of different diameter. If the volume of a modified cylinder is V′, then we define

$$V' = \pi a'^2 \, 2c \; . \tag{3}$$

With the volume of the original cylinder

$$V_c = \pi a^2 \, 2c \quad ,$$

the new fineness ratio, therefore, is:

$$c/a' = c/a \sqrt{\frac{V'}{V_c}} \quad . \tag{4}$$

With this fineness ratio, the corresponding fill-ratio for a resonant frequency is to be found from Stewartson's tables. However, the fill-ratios and hence the fineness ratios, computed by the above procedure, were found to be overestimated relative to the observed values. Moreover, the discrepancy increased systematically with the height of the conical cup, h/2c . A correction, therefore, was necessary. This correction to the fill-ratio, as inferred from an "equivalent cylinder" computed as outlined above, was found to be:

$$\text{Correction:} \quad \Delta = -\,22(h/2c - .25) \text{ percent} \tag{5}$$

which should be added, algebraically, to an "equivalent cylinder" value. For conical cups, therefore, of heights one quarter of the length of the

cylinder, or less, no correction is necessary. When corrections were applied to longer cups, the agreement with the observations was improved markedly. The results are shown in Table VI. The prediction appears to be poorest for 2° angles but is within one to two percent elsewhere.

4.3 Other Modifications

Some other modifications were tested also, and the results are given in Table IV. Previous tests with conical cups were made with a square base at one end, $R/a = 0$. If a rounded base is added to the conical cup (configuration F_1 and F_2) an additional shift in frequency may take place. Unfortunately, these tests are rather fragmentary and no definite conclusion can be made safely at this time. On the whole, the addition of a rounded base to a conical cup produces no additional significant change in the fill-ratio, provided that $R/a \leq 0.6$. For $R/a > 0.6$, the effect is approximately additive, i.e., the effect due to the conical cup and the rounded base act separately. A combination of two conical cups, C and E_1 , also suggests that the effects of separate modifications are additive.

One test was made with a cavity, $c/a = 3.08$, but with its walls defined by a circular arc of radius $R = 10$ inches tangent to the midpoint of the cavity. No resonance was found. One-half of this two-cup combination was then tested in a cylinder, $c/a = 2.69$, configuration E_2. Treating this cavity by an "equivalent cylinder" method led to the prediction of the fill-ratio for resonance at 66 percent, whereas the observed value is 69 percent. This cup also was tested in combination with rounded corners. All resonances were rather weak and had double peaks for $R/a = 0$ and $R/a = 0.4$

TABLE VI

Prediction of Fill-Ratio for Resonance by an "Equivalent Cylindrical Cavity", ECC, with a Correction

$c/a = 3.077$

Configuration C

$\frac{h}{2c} = .26$

θ°	ECC,%	Obs,%
2	85	87
5	90	90
7	92	92
$8\frac{1}{2}$	94	94

Configuration D

$\frac{h}{2c} = .46$

θ°	ECC,%	Obs,%
2	90	86

$c/a = 2.687$

$\frac{h}{2c} = .30$

θ°	ECC	Obs
2	60	61
5	62	62
$8\frac{1}{2}$	64	64
12	66	67

$\frac{h}{2c} = .53$

θ°	ECC	Obs
2	58	61
$5\frac{1}{2}$	66	64
$7\frac{1}{2}$	70	70
$8\frac{1}{2}$	73	73
$9\frac{1}{2}$	75	76

Configuration E_1

$\frac{h}{2c} = .75$

θ°	ECC	Obs
3	63	65
4	69	67

Combination C and E_1

$\frac{h}{2c} = 1.0$ $\quad E_1$: $\theta = 3^\circ$

C	ECC	Obs
$\theta^\circ = 7^\circ$	64	64
$8\frac{1}{2}$	68	67

4.4 Search for Higher Resonance Modes

In view of the appearence of double resonance peaks in some configurations, the presence of higher resonance modes was carefully examined. None was found that could account for the observed doubling.

It was of interest, however, to pursue this matter further. With a very sensitive gyroscope, there was a chance of detecting the higher resonance modes in the two cylindrical cavities, c/a = 3.08 and 2.69. If one designates the columns in Stewartson's tables by k = 1, 2, 3 (radial modes) and the rows by j (axial modes) where j is its value in $\frac{c/a}{2j+1}$, then the observed principal frequency in the two cavities is j = 1 and k = 1 or (1,1). In these cavities the j = 2 is absent, but j = 3 and 4 are theoretically present at various fill-ratios as shown in Table VII.

TABLE VII
Resonances at Percentage of Fill

	c/a = 3.077			c/a = 2.687		
k	1	2	3	1	2	3
j						
0	—	—	—	—	—	—
1	84.5 (.522)	—	—	60.3 (.364)	—	—
2	—	—	—	—	—	—
3	—	84. (.029)	—	—	75 (.031)	—
4	—	—	67 (.017)	—	60 (.015)	92 (.010)

The numbers in parentheses are the associated rates at which the nutational component of amplitude would be expected to diverge. The numbers associated with the (1, 1) modes are the observed values. Although the gyroscope was sensitive enough to detect such instabilities, none was found. This suggests that in higher modes the effect of viscosity in damping the oscillations is even stronger than in the principal mode. The mode (2,1) was previously observed in Reference 3 with a cylinder $c/a = 5.16$. In the present cylinder, $c/a = 3.08$, the (3,2) mode almost coincides with the (1,1) mode, but no indication of asymmetry could be detected. Likewise, in the $c/a = 2.69$ cylinder, where there is a clear separation of the (3,2) and (1,1) modes, the former could not be detected. Also in a smaller cylinder, $c/a = 2.230$, especially designed to have the (2,2) mode as its principal frequency, only a suggestion of instability, $\alpha_1 \approx 0.005$, could be detected at the predicted resonant fill-ratio. All of these observations seem to suggest that only the first column in Stewartson's tables has any practical relevance unless the fluid is exceptionally heavy.

4.5 Central Rod

As previously stated, if the shell cavity contains a central column, it should not be filled to more than 96 percent of the available volume. At fill-ratios higher than 96 percent, the column interferes with the distributed free surface of the liquid at resonance, and the application of Stewartson's tables to such configurations becomes invalid. On the other hand, if a cavity with an air space b^2/a^2 happens to have a resonant

frequency leading to instability, the insertion of the central column of diameter $2r \approx 2b$ may suppress this instability. Such an operation, however, is rather risky. Evaporation of the fluid may depress the fill-ratio below 96 percent thereby returning the system to instability.

5. CONCLUDING REMARKS

An exploratory survey with a gyroscope of the effect of modifications of the cylindrical cavity on frequencies showed a significant shift in the fill-ratio at which the principal frequency occurred in modified cavities. However, the absence of theoretical guide lines, similar to those for the cylindrical cavity, makes prediction of resonance in a non-cylindrical cavity an uncertain procedure except for configurations covered by Wedemeyer's analysis, (see Appendix).

Tentatively, the following simple rules may be used in practice. The effect of rounding the corners of the cylindrical cavity can be disregarded for $R/a \leq 0.6$. For a nearly hemispherical base, the shift in the fill-ratio for resonance can be estimated either from Figure 2, or by an "equivalent cylinder" method with a correction, as outlined in Section 4.1 .

For conical or nearly conical constriction of the circular cylinder, the effect can be estimated as outlined in Section 4.2. The basic cylinder should be taken as defined by the maximum diameter of the cavity, 2a , and a maximum height, 2c. If the burster of the fuze protrudes into the cavity, an estimate of maximum height may present practical difficulties. In such cases some judgement must be exercised. It is

suggested that if the burster is short and occupies most of the volume of the cavity to its depth, the maximum height of the cavity, 2c, should be measured to the burster. At any rate, maximum diameter and maximum height will define the cylindrical volume, V_c. The actual volume is to be replaced by an "equivalent cylinder" of the same height, 2c, but of different diameter, 2a′. The fill-ratio for resonant frequency is to be found with the new fineness ratio, c/a′, from Stewartson's tables. The resulting fill-ratio, in percent, is to be corrected by adding, algebraically, - 22 (h/2c - .25), where h is the height of the conical section of the non-cylindrical cavity. If the cavity has no sharp breaks in its curvature that may define the height of the "conical section", h/2c, an estimate can be made from the drawing by replacing the curved boundary with the conical section enclosing the same volume. The angle of the generatrix of the truncated cone should correspond to the average angle of the curved surface.*

The designer should note some beneficial effects of non-cylindrical cavities. The instability, at resonance, is usually less than in corresponding cylindrical cavities. If the departure from the cylindrical cavity is large enough, the instability might become sufficiently mild to be controllable by the aerodynamic damping. Also, the shift in the resonant frequency in a modified cavity may be large enough to remove it from consideration at the prescribed fill-ratio. As an example, consider the cavity c/a = 3.08, configuration D, with $\theta = 4^{\circ}$, in Figure 5.

*However, see Appendix for more rational way of handling the predictions for certain cavity shapes.

This configuration is stable at all fill conditions, whereas the parent cavity is strongly unstable at 85 percent fill-ratio.

Also, in practice, only the principal radial mode ($k = 1$) needs to be considered; all higher modes may be neglected.

At present, it is impossible to say how well the suggested simple rules will apply to modifications different from those tested, or even to homologous cavities but of different diameters. For the latter, it is planned to make a few spot checks in a 2 inch diameter cavity. It is anticipated, however, that the established trends, except for possible Reynolds number effects, will hold in smaller cavities. At present, nothing can be said from experimental evidence about the effects of significantly different types of modifications.

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The author is very grateful to Mr. E. H. Wedemeyer for writing the Appendix so as to expedite the availability of the results of his theoretical investigation on the effects on eigen frequencies of modifications of the cylindrical cavity. Full account of his analysis will eventually be published separately.

B. G. KARPOV

REFERENCES

1. Stewartson, K. On the Stability of Spinning Top Containing Liquid. J. Fluid Mech., 5, Part 4, 1959.

2. Wedemeyer, E. H. Dynamics of Liquid Shell: Theory of Viscous Corrections to Stewartson Stability Problem. BRL Report No. 1287, June 1965.

3. Karpov, B. G. Dynamics of Liquid-Filled Shell: Resonance and Effect of Viscosity. BRL Report No. 1279, May 1965.

4. Karpov, B. G. Liquid Filled Gyroscope: The Effect of Reynolds Number on Resonance. BRL Report No. 1302, October 1965.

5. Karpov, B. G. Dynamics of Liquid-Filled Shell: Aids for Designers. BRL Memorandum Report 1477, May 1963.

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APPENDIX

By the time the present report was completed, Mr. Wedemeyer had found a rational way, based on theoretical considerations, of estimating eigen frequencies in non-cylindrical cavities. These results will, eventually, be published separately. Meanwhile, he has kindly consented to give a brief summary of the rules of the game for immediate benefit to the designer.

By E. H. Wedemeyer

Without derivation, a formula is given which allows approximate evaluation of the eigen frequencies, τ, for a given fill-ratio, or the fill-ratio for a given τ for certain non-cylindrical cavity shapes. The approximation is based on the assumption that the radius of the cavity, $a(z)$, changes slowly as a function of distance along the axis, i.e., $\left|\frac{da(z)}{dz}\right| \ll 1$ everywhere.

In Stewartson's tables the fineness ratio, or rather $\frac{c}{a[2j+1]}$ is tabulated as function of b^2/a^2 and τ. Let us call this function $S(b^2/a^2, \tau)$. Let us assume that the radius of the cavity is given as a function of z; i.e., $a(z)$ for $0 < z < 2c$ and that $\left|\frac{da}{dz}\right|$ is small everywhere. Then, for resonance, the following relation can be shown to be approximately valid:

$$1 = \frac{1}{2c} \int_0^{2c} \frac{c/a[2j+1]}{S(b^2/a^2, \tau)} dz \quad . \tag{1}$$

The radius, a, which occurs two times in the integrand of Equation (1)

(namely in c/a and in the argument b^2/a^2 of the function S) is to be taken as the local, z - dependent radius a(z).

If the cavity is 100 percent filled, Equation (1) simplifies to:

$$S(0,\tau) = \frac{1}{2c}\int_0^{2c} \frac{c}{a[2j+1]}\,dz \quad . \tag{2}$$

Equation (2) can be interpreted in the following way: The right side of Equation (2) gives an "effective" fineness-ratio or rather an effective $\frac{c}{a[2j+1]}$ in the sense that a cylindrical cavity having a $\frac{c}{a[2j+1]}$ - value equal to the right side of (2) will have the same eigen frequencies as the non-cylindrical cavity.

Equation (2) allows determination of the eigen frequencies τ just by performing the integration for $S(0,\tau)$ and looking up the corresponding τ in Stewartson's tables.

For a partially filled cavity, Equation (1) can be interpreted as either that the τ is to be determined for a given radius b of the void, or b is to be determined for a given τ . In either case, the evaluation is troublesome since $S(b^2/a^2,\tau)$ is given only as a tabulated function and in order to solve the integral one must assume both τ and b and eventually repeat the integration with changed values of τ (or b) until, by interpolation, the correct value of τ (or b) is obtained.

The simplicity obtained when the cavity is completely full suggests seeking an approximate evaluation of (1) in cases where the cavity is almost completely full (these cases are the most frequent in practice).

One can then try to expand $1/S$ into a power-series in b^2/a^2 with coefficients dependent on τ . If τ is given and the problem is to determine b, the most accurate way is to plot $1/S$ as given by Stewartson's tables for the particular τ versus b^2/a^2, and approximate the curve by a power-series. If neither τ nor b^2/a^2 are too large, the following formula is quite handy and is a good approximation:

$$\frac{1}{S} = \frac{1}{S_o} \left[1 + 1.26 \left(\frac{b^4}{a^4}\right)\right] \quad . \tag{3}$$

So is the value $S(0,\tau)$ as obtained from Stewartson's tables for $b^2/a^2 = 0$.

The approximation (3) is quite good within the limits

$$0 < \tau < 0.12$$
$$0 < b^2/a^2 < 0.15 \tag{4}$$

with (3) substituted into (1), one obtains:

$$S_o(\tau) = \frac{1}{2c} \int_0^{2c} \frac{c}{a[2j+1]} \quad . \quad \left[1 + 1.26\left(\frac{b^4}{a^4}\right)\right] dz \quad . \tag{5}$$

The advantage of Equation (5) is that the right side is independent of τ, i.e., by integrating (5) without fixing the value of b one obtains an explicit relation between τ and b, which is of the form:

$$S_o(\tau) = c_1 + c_2 \quad . \quad b^4 \tag{6}$$

where $c_1 = \frac{1}{2c} \int_0^{2c} \frac{c}{a[2j+1]} \, dz$ and $c_2 = \frac{1.26}{2c} \int_0^{2c} \frac{c}{a^5 [2j+1]} \, dz$.

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13. ABSTRACT

A search was made, using a liquid-filled gyroscope, for resonance frequencies in modified cylindrical cavities. The fill-ratio at which the resonant frequency occurs shows considerable sensitivity to cylindrical modifications. Simple rules are given for predicting, approximately, the change in this fill-ratio as a function of the modifications. In the Appendix, Wedemeyer suggests a better way of prediction.

Tests of a cylindrical cavity with a central rod show that the air gap between the column and the free surface of the disturbed fluid should not, at small yaws, be less than 1/16 inch for non-interference with resonant conditions.

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